BPhO Round 1 marking 2017

- Positive marking is the aim. Marks should be awarded for good physics, even if the reasoning does not follow the mark scheme. Alternative routes to the answers can be allowed.
- Significant figures. A leeway of ±1 sig fig is allowed; if the published solution gives 3, allow 2 or 4 in the students answer. This only applies to the final answers as intermediate answers may be recorded to greater precision to avoid rounding errors. Candidates should not lose more than 1 mark per question for this even if they have got it wrong in more than one place in the question (they might lose all their hard earned marks otherwise). Question 1 (a) is different in this respect as it is specifically about sf and dp.
- Units should be given for the final answer. It may be that the unit is given a little earlier and that it does not appear on the very last line. Some allowance may be made if it is clear that the unit has been used a line or two earlier.
- In one or two places the units are a required part of the answer for the mark, and so must be there.
- Error carried forward (ecf) is allowed provided ridiculous results do not start appearing. A mark is lost for the initial mistake, but then they can carry on (if it is possible) to gain some of the subsequent marks.
- You are not required to spend time deciphering scribble.

- In some cases, the mark scheme allows for different methods of solution. You will need to award marks for these solutions, with a mark for each key step in heading for a solution. You cannot give more marks than is available.
- The significant figures – although some details are given above, this is not a paper testing use of significant figures. Two sig figures in the answer to almost every question would be fine. I think you will find some inconsistencies in the solutions, with more figures given than might be justified.
- There are a few questions in which two or at most three marks are awarded for working, whose details are not shown. For example, Qu 3 on the crane; there are a number of ways of getting the results, but students will generally not set out their path. Therefore, a mark for a sensible resolving, or taking moments about a point, etc. will contribute to the working marks; often students will get lost. You do not need to find the exact point at which they made a mistake. Wrong answer – they lose that mark. Some working heading in the right direction, one or two marks. Judgement is required, but not timewasting.

- If stuck, email robin Hughes on rh584@cam.ac.uk and I will reply fairly quickly.

BPhO Round 1 2017.

Section 1.

Qu 1 (a) $1\ \dfrac{\text{foot}}{\text{ns}} = \dfrac{1}{3}\ \dfrac{\text{yd}}{\text{ns}}$

$= \dfrac{1}{3} \times \dfrac{1}{1.094}\ \dfrac{\text{m}}{\text{ns}}$

$= 3.047 \times 10^{8}\ \text{m s}^{-1}$ ✓

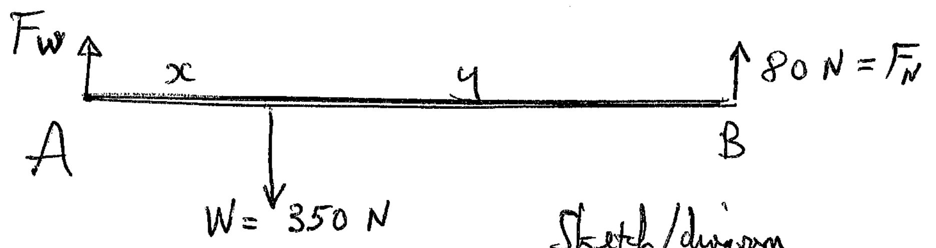
Error $= \dfrac{3.047 - 3.000}{3.000}$ ✓

$= 1.564\ \%$

$= \underline{1.6\ \%}$ ✓ (3)

* $\dfrac{1 - 0.9846}{3} = 1.54\%$ only gains $\frac{2}{3}$ marks

(b)
(i)



Sketch/diagram ✓

Moment about A ↺ $5 \times 80 - 350.x = 0$ ✓

$x = \dfrac{400}{350} = \dfrac{8}{7}\ \text{m}$ ✓

$= \underline{1.14\ \text{m}}$

(ii) $x + y = 5.0\ \text{m}$

$y = \dfrac{27}{7}\ \text{m} = 3.86\ \text{m}$ ✓

Moments about B ↺ $F_W \times 5 - 350 \times \dfrac{27}{7}$

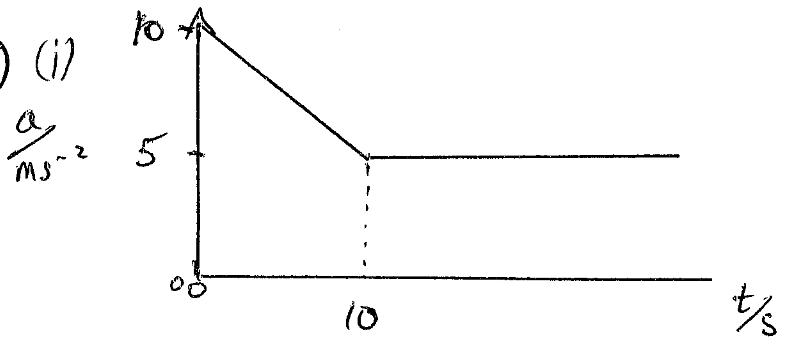
$\underline{F_W = 270\ \text{N}}$ ✓

[or $F_W + 80 = 350$ ✓
$\Rightarrow F_W = 270\ \text{N}$ ✓].

(5)

Answers as fractions, or to 2 s.f. are o.k.

(c) (i)



Graphically.

change in velocity = area under a-t graph

In first 10 s, average acceleration is $7.5\ ms^{-2}$

So change in velocity is $75\ ms^{-1}$ ✓

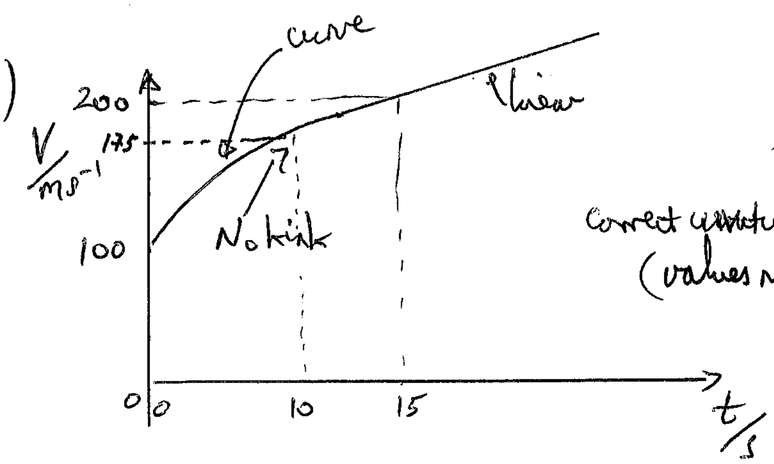
So, After 10 s, the velocity is $100 + 75 = 175\ ms^{-1}$

To reach $200\ ms^{-1}$, an extra $25\ ms^{-1}$ is needed ✓

Constant acceleration of $5\ ms^{-2}$ would take 5 s ✓

which is 15 s after the start ✓

(ii)



shape ✓

no kink ✓

correct curvature + linear ✓

(values not required).

(7)

or algebraically

$$a = -\tfrac{1}{2}t + 10 \quad (\text{in SI units})$$

$\frac{da}{dt}$

$$\int_{100}^{V} a\,dt = \int_0^{10} \left(-\tfrac{1}{2}t + 10\right) dt$$

$$V - 100 = \left. \frac{-1t^2}{4} + 10t \right|_0^{10}$$

$$= 75\ ms^{-1}$$

$$V = 175\ ms^{-1}$$

4 marks

or by comparison with "$s = ut + \frac{1}{2}at^2$" (const a)

$$v - u = at + \frac{1}{2}\left(\frac{da}{dt}\right) \cdot t^2 \quad (\text{const. } \tfrac{da}{dt} \text{ for } 10\text{ s})$$

$$v - 100 = 10 \cdot 10 + \frac{1}{2}\left(-\frac{1}{2}\right) 10^2$$

$$= 100 - 25$$

(4 marks)

$$\underline{v = 175 \text{ ms}^{-1}}$$

or integrate from the start., (4 marks)

$$v_f = 200 = 100 + \int_0^{10} (-kt + 10)\,dt + \int_{10}^{t} 5\,dt$$

$$\frac{da}{dt} = k = \frac{(\cancel{5} - \cancel{10})}{5} \quad \therefore \quad v_f = 100 \left(-\frac{1}{2}\frac{t^2}{2} + 10t\right)\Big|_0^{10} + 5(t - 10)$$

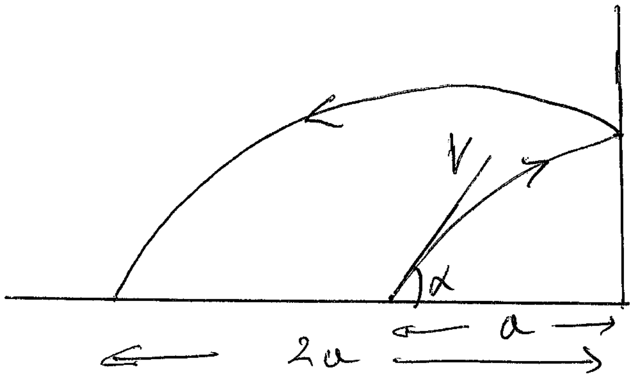
$$= -\frac{1}{2} \text{ ms}^{-2}$$

$$200 = 100 - 25 + 100 + 5t - 50$$

$$\underline{t = 15 \text{ s}}$$

(4)

(d)



Horizontal components : $V_{before} = V\cos\alpha$ ✓

$V_{after} = e\,V_{before} = e\,V\cos\alpha$ ✓

So, distance $a = t_1 \cdot V\cos\alpha$ (time of flight to right, t_1)

and $2a = t_2 \cdot e\,V\cos\alpha$ (" " " to left, t_2) ✓

Hence $t_2 = \frac{2}{e} \cdot t_1$ ($t_2 = 3t_1$) ✓

The vertical velocity is unaffected by the collision. ✓

So, total time in flight is $2 \times \frac{V_{vertical}}{g}$ ✓

i.e. $t_1 + t_2 = \frac{2V\sin\alpha}{g}$

$$\therefore \frac{2V\sin\alpha}{g} = \frac{a}{V\cos\alpha} + \frac{2}{e} \cdot \frac{a}{V\cos\alpha}$$

$$\frac{2V^2\sin\alpha \cdot \cos\alpha}{g} = \left(\frac{2}{e} + 1\right)a$$

"4" (6)

$$V^2 \sin 2\alpha = \left(\frac{2}{e} + 1\right) g \cdot a$$

$$V^2 \sin 2\alpha = 4ga$$ ✓

Other ideas : V $V\sin\alpha = \frac{1}{2}gt$ t = time of flight (no effect of wall)

H. $V\cos\alpha \cdot t_1 = a$

and $\frac{2}{3} V\cos\alpha \cdot t_2 = 2a$

knowing $t = t_1 + t_2 = \frac{2a \cdot 3}{2V\cos\alpha} + \frac{a}{V\cos\alpha} = \frac{4a}{V\cos\alpha}$

(5)

(e)

Weight of helicopter, $W = 9800\,N$ ✓

Required for the marks either a statement or a recognisable formula version.

Force (lift) = rate of change of momentum. ✓

$$\frac{\Delta p}{\Delta t} = \frac{\Delta (mv)}{\Delta t} = v\frac{\Delta m}{\Delta t}$$

$$m = \rho V$$

$$\frac{\Delta p}{\Delta t} = v\rho\frac{\Delta V}{\Delta t}$$

$$= v^2\rho A$$ ✓

$$W = v^2\rho A$$ ✓

so $$v^2 = \frac{W}{\rho A} = \frac{9800}{1.2\times10^3\,\pi\times3^2}$$

$$= 288$$

$$\underline{v = 17\,ms^{-1}}$$ ✓

(5)

6 A

(f)

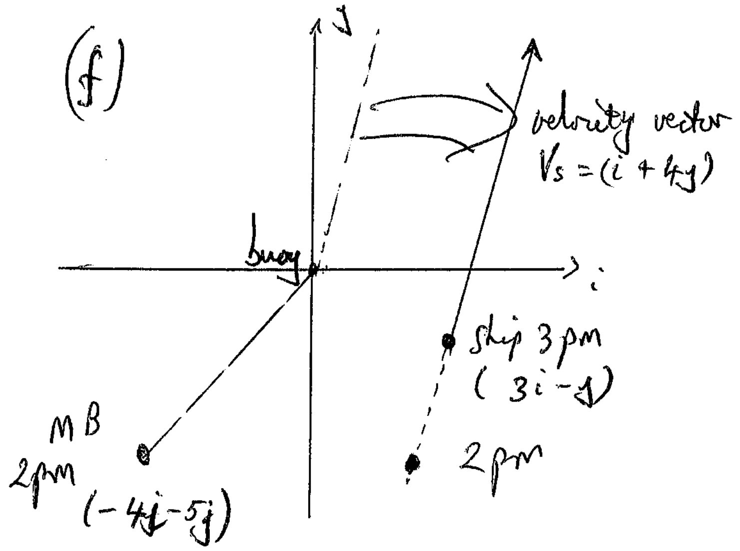


Diagram ✓

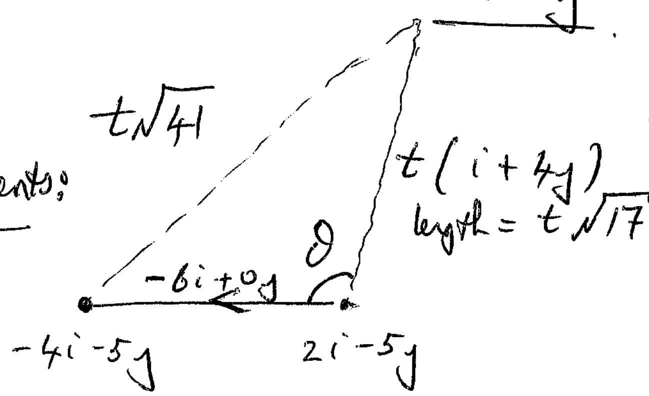
(i) Position of ship at 2pm is vector point at 3pm − 1 hour × V_s ✓

$= 3i - j - 1(i + 4j)$

$= \underline{2i - 5j}$ ✓

(ii)

Displacements:



* (✓✓) − 2 marks for clear working to obtain t and V_{MB}. If not clear give 1 or even 0 marks for the working. But give the marks for each answer.
So, no working 7 → 5.

Using the cosine rule, $t^2 41 = 36 + t^2 17 - 2t\sqrt{17}\cdot\sqrt{36}\cdot\cos\theta$

and $\cos\theta = \frac{a \cdot b}{|a||b|} = \frac{(-6i + 0j)\cdot t(i + 4j)}{\sqrt{36}\; t\sqrt{17}}$

$= \frac{-6t}{\sqrt{36}\, t\sqrt{17}}$

$= -\frac{1}{\sqrt{17}}$

$\therefore \quad 41t^2 = 36 + 17t^2 + 2t\sqrt{17}\cdot\sqrt{36}\cdot\frac{1}{\sqrt{17}}$

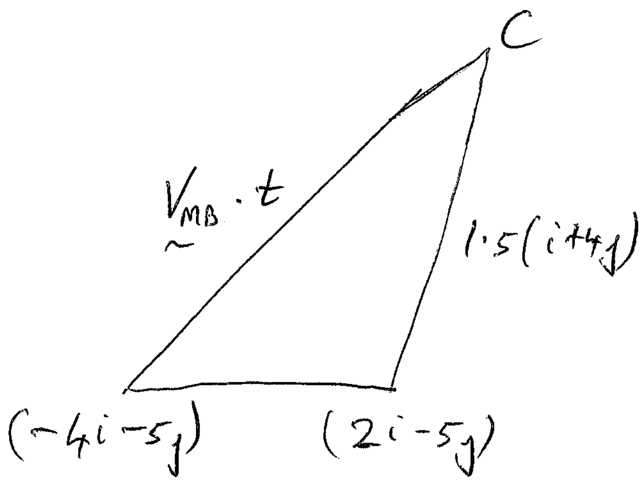
$24t^2 = 36 + 2\times 6t$

$2t^2 - t - 3 = 0$

$t = \frac{1 \pm \sqrt{1 + 24}}{4} = \frac{1}{4} \pm \frac{5}{4} \Rightarrow \underline{t = 1.5h}$ ✓

i.e. interception at 3:30 pm

6 B



point C is at; $2i-5j + 1.5(i+4j)$
i.e. $(3.5i + j)$

So that $V_{MB} \times 1.5 = (3.5i + j) - (-4i-5j)$

$$V_{MB} \times 1.5 = 7.5i + 6j$$
$$V_{MB} = 5i + 4j$$

check $|V_{MB}| = \sqrt{5^2+4^2}$
$= \sqrt{41}$. as given in question.

7

or $V_{MB} = ai + bj$.
Ship and boat end up at the same point after time t.
$\therefore$ Add the respective velocities to the 2pm start points.

Hence $-4i-5j + \underset{(MB)}{t(ai+bj)} = 2i-5j + \underset{\text{ship}}{t(i+4j)}$

Giving $t(a-1)i + t(b-4)j = 6i + 0j$

equating for i: $t(a-1) = 6$
" " j: $t(b-4) = 0 \Rightarrow b = 4$

but we know (given) $|V_{MB}| = \sqrt{41}$
So $a^2+b^2 = 41$ with $b=4$.
$a = \pm 5$. But t is positive so $a = +5$
$V_{MB} = \underline{5i+4j}$, with $\underline{t = 1.5h}$ (+2pm)

(g) $\rightarrow 60°C \rightarrow 38°C$

For water : $\Delta Q_W = mc\Delta T$

$\Delta Q_W = 1 \times 4200 \times (60-38)$

$= 92400 \text{ J}$ ✓

Steam $\rightarrow$ 100°C $\xrightarrow[\text{at } 100°C]{\text{Water}}$ 82°C $\rightarrow$

For condensing steam,

$\Delta Q = mL + mc\Delta T$

$= m(L + c\Delta T)$ ✓

Require, $92400 = m(2.26 \times 10^6 + 4200(100-82))$ ✓

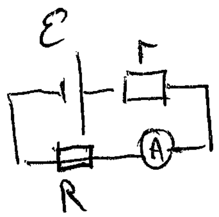
* If $mc\Delta t$ term left out to give 40.9g then lose 1 mark.

$m = \dfrac{92400}{(2.26 \times 10^6 + 4200 \times 18)} = \dfrac{92400}{2.26\times10^6 \;\; 75600}$

$m = 39.6 \text{ g} = 40 \text{ g} = 0.040 \text{ kg}$ ✓

(4)

(h)



$\mathcal{E} = Ir + IR$ ✓ equation in symbols or values.

For $R = 2.0\,\Omega$, $\mathcal{E} = 0.8r + 0.8 \times 2$

$= 0.8r + 1.6$ ✓ simultaneous equations.

For $R = \frac{10}{7}\,\Omega$, $\mathcal{E} = 1.0r + \frac{10}{7}$

$R_{\parallel}$: $\dfrac{1}{R_{\parallel}} = \dfrac{1}{5} + \dfrac{1}{2}$

$R_{\parallel} = \dfrac{10}{7}\,\Omega$ ✓

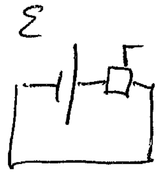
$= 1.4(3)\,\Omega$

Subtracting, $0 = -0.2r + 1.6 - \frac{10}{7}$

$r = \frac{6}{7}\,\Omega = 0.86\,\Omega$ ✓

$\mathcal{E} = \frac{16}{7} \text{ V} = 2.3 \text{ V}$ ✓

(5)

(i)  shorted. (8)

$I = \frac{\varepsilon}{r} \Rightarrow 3 = \frac{6}{r}$

so $\underline{r = 2\,\Omega}$ ✓

Maximum power dissipated externally in R, is when $R = r$ ✓
or differentiate to see this result.

$\frac{d}{dR} P_R = \frac{d}{dR} \frac{\varepsilon^2 R}{(R+r)^2} = 0$

$\Rightarrow R = r$

or give the result.

Then $I = \frac{\varepsilon}{r+R} = \frac{6}{4} = 1.5\,A$

$$\text{Power} = I^2 R = 1.5^2 \times 2 = 4.5\,W$$ ✓

Energy dissipated externally in R in 1 minute is 60×4.5
$= \underline{\underline{270\,J}}$. ✓

(4)

(j) $f = \frac{c}{\lambda} = \frac{3 \times 10^8}{639 \times 10^{-9}} = 469.5 \times 10^{12}\,Hz$.

$E_{photon} = hf = 6.63 \times 10^{-34} \times 469.5 \times 10^{12}$

$= 3.11 \times 10^{-19}\,J$ ✓

$N_{photons}$ per nanosecond $= \frac{\text{power} \times \text{time}}{E_{photon}}$ ✓

$= \frac{0.5 \times 10^{-3} \times 10^{-9}}{3.11 \times 10^{-19}}$

$= \underline{\underline{1.6 \times 10^{6}}}$ ✓

(3)

(k)

NII. $T_{top} + mg = \frac{m V_{top}^2}{r}$ (inwards) ✓

$T_{top} = 0 \Rightarrow mg = \frac{m V_{top}^2}{r}$

$V_{top} = \sqrt{rg}$ ✓

$T_{bott.} - mg = \frac{m V_{bottom}^2}{r}$ ✓

Energy Cons. $\frac{1}{2} m V_{top}^2 + mg\,2r = \frac{1}{2} m V_{bottom}^2$ ✓

$V_{bottom}^2 = V_{top}^2 + 4gr$

$= rg + 4rg$

$= 5rg$ ✓

$\therefore T_{bottom} = m \cdot \frac{5rg}{r} + mg$ R if wrong but correctly added give this final mark.

$= \underline{6mg}$.

(6)

(l) For SHM, $x = A \sin(\omega t)$

$V = A\omega \cos(\omega t)$

$a = -A\omega^2$

Sand loses contact when the <u>minimum</u> value of the acceleration is equal to g. ✓

$\therefore g = A\omega^2$ ✓

$f_{min}^2 = \frac{g}{4\pi^2 A}$ ✓

$f_{min}^2 = \frac{9.81}{4\pi^2 \times 2\times10^{-4}}$, $\underline{f_{min} = 35(.2)\ Hz}$.

(3)

(10)

(m)

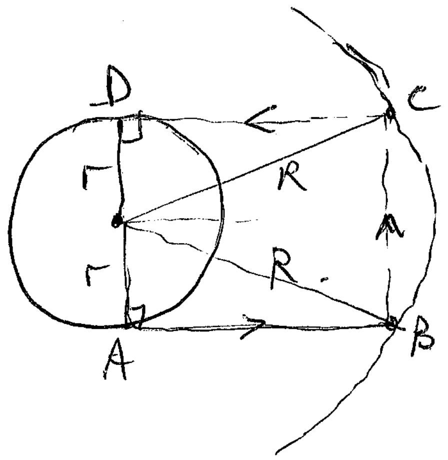


diagram ✓

tangential paths. ✓

$R = 3.59\times10^4 + 6370$ km
$= 4.227\times10^4$ km

$ABCD = l = 2\sqrt{R^2 - r^2} + 2r$ ✓
(AB + DC)   (BC)

$l = 2\sqrt{4.28^2\times10^8 - 6370^2} + 2\times6370$ km
$= 2 \times 41.8\times10^3 + 2\times6370$ km
$= 9.63 \times 10^4$ km
$= 9.63 \times 10^7$ m ✓

If R taken as 3.59×10^4 km then
$l = 2\sqrt{3.59^2\times10^8 - 6370^2} + 2\times6370$
$= 8.34\times10^4$ km
$= 8.3\times10^7$ m.
with $\Delta t = 0.28$ s

$\Delta t = \frac{l}{c} = \frac{9.63\times10^7}{3\times10^8}$ ✓

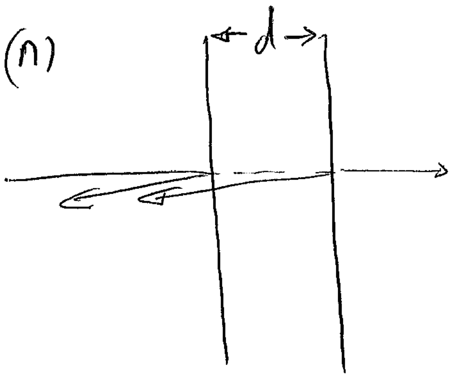
$= 0.32(1)$ s ✓

Max. of 5 marks total.

(6)

(11)

(n)



For constructive interference, path difference $= m\lambda$
(m integer)

There will be a phase difference of π at each reflecting surface, which means that there is no phase difference introduced overall. ✓

$\therefore \quad 2d = m\lambda$

λ is decreased in the medium by a factor $\frac{1}{n}$. ✓

Hence $\quad \lambda = \frac{n\,2d}{m}$ ✓

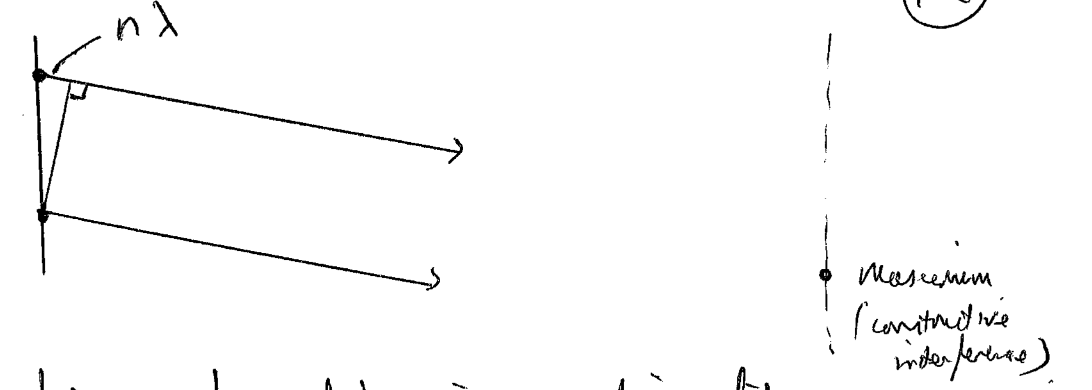
For $m = 1$, $\quad \lambda = \frac{1.52 \times 2 \times 0.42 \times 10^{-6}}{1.}$

$= 1.28 \times 10^{-6}$ m $= 1280$ nm ; ✓
IR.

For $m = 2$, $\quad \lambda = 638$ nm. ✓
Visible.

(5).

(a) 12



A path difference change of λ gives one fringe shift.
" " " " 25λ " 25 fringe shift ✓

In the 5 cm tube, there are $N\lambda$ length of wave. ✓
In the vacuum in the tube there are $(N-25)\lambda$ length of wave. ✓
as the wavelength is ~~shorter~~ in air, longer in a vacuum.
(speed of light is slightly less in air).

$$\text{Optical path is} \quad (N-25)\lambda_{vac} = N\lambda_{air} = N\frac{\lambda_{vac}}{n}$$ ✓ optical path idea

$$\therefore \quad N - 25 = \frac{N}{n}$$

$$n = \frac{N}{N-25}$$

$$\text{and} \quad N = \frac{5\times10^{-2}\ \text{m}}{600\times10^{-9}\ \text{m}} = 8.333\times10^{4}$$

which gives $n = 1.0003 \quad (= 1 + 3\times10^{-4})$ ✓

Or

$$25 \text{ fringes} = \frac{5\text{ cm}}{\lambda/n} - \frac{5\text{ cm}}{\lambda}$$

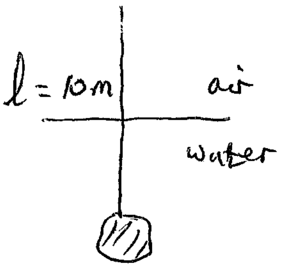
$$= \frac{5\times10^{-2}}{600\times10^{-9}}(n-1)$$

$$3000\times10^{-7} = n-1$$

$$\underline{n = 1.0003}$$

(5)

(p)



$A = 5.0\ cm^2$

Young: $E = \frac{F}{A} \cdot \frac{l}{\Delta l} \Rightarrow \Delta l = \frac{Fl}{E.A}$

In air, $\Delta l = \frac{mg.l}{E.A}$ correct equation or values (see below) ✓

In water, upthrust is given by $\rho_w \cdot V_{sub} \cdot g$ (weight of water displaced). ✓

ρ_w = density of water
ρ_{sub} = density of submerged wreck.

$$\text{Upthrust} = \rho_w \cdot \frac{M_{sub}}{\rho_{sub}} \cdot g$$ ✓

Since $\Delta l \propto$ tension,
the change in extension Δe is $\propto$ change in load i.e. the upthrust.

$$\text{so } \Delta e = \left(\frac{\rho_w}{\rho_{sub}} \cdot M_{sub} \cdot g\right) \cdot \frac{l}{E.A}$$ ✓

$$= \frac{1000}{8000} \times 1 \times 10^4 \times 9.81 \times \frac{10}{5 \times 10^{10} \times 5 \times 10^{-4}}$$

$$= \underline{4.9 \times 10^{-3}\ m}$$ ✓

(5)

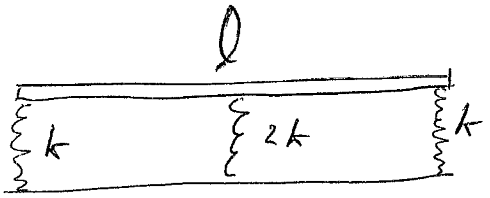
Or

load in air = 9.81×10^4 N ✓
extension in air = 39.24×10^{-3} m ✓
extension in water = extension in air − upthrust ✓
= 34.3×10^{-3} m ✓
difference = $\underline{4.9 \times 10^{-3}\ m}$ ✓

Volume of wreck = $1.25\ m^3$
Upthrust = 12,250 N
Weight = 98,000 N

(14) A.

(9) (i)



Symmetric arrangement, so level beam ✓

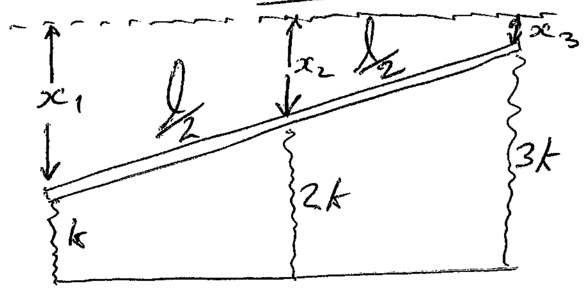
"compressive force $= kx$"

Total load, so $Mg = kx + 2k.x + kx$ ✓

$= 4kx$

Thus, $x = \dfrac{Mg}{4k}$. $\quad$ $\left.\begin{array}{l} F_{left} = F_{right} = \dfrac{Mg}{4} \\ F_{centre} = \dfrac{Mg}{2} \end{array}\right\}$. ✓

(ii)



Total load, $Mg = k.x_1 + 2k.x_2 + 3k.x_3$ ① ✓

Relate x_1, x_2, x_3.

Board is linear : $x_1 - x_3 = 2(x_2 - x_3)$ ✓

$2x_2 = x_1 + x_3$ ②

Take moments about centre : $\cancel{\frac{M}{2}g\frac{l}{4}} - \frac{l}{2}.kx_1 - \cancel{\frac{m}{2}.g\frac{l}{4}} + \frac{l}{2}3k.x_3 = 0$

(+)

So, $x_1 = 3x_3$ ③ ✓

Eliminating x_1 in ② and ③, $x_2 = 2x_3$ ④

So $Mg = k.3x_3 + 2k.2x_3 + 3kx_3$

Here you need values of x_1, x_2, x_3 not F_1, F_2, F_3 since $F_1 = F_2 = F_3$.

$\left.\begin{array}{l} x_3 = \dfrac{Mg}{10k} \\ x_2 = \dfrac{2}{10}.\dfrac{Mg}{k} \\ x_1 = \dfrac{3}{10}.\dfrac{Mg}{k} \end{array}\right\}$. ✓ $\quad$ $\left.\begin{array}{l} F_3 = \dfrac{3}{10}Mg \\ F_2 = \dfrac{4}{10}Mg \\ F_1 = \dfrac{3}{10}Mg \end{array}\right\}$. ✓

$(F_1 + F_2 + F_3 = Mg)$.

(8)

14B

(9) continued.

Equations needed are.

equ. (1) total load
(2) Beam is linear
(3) moments about centre.

or
- moments about LH end: $m\frac{g}{2} = kx_2 + kx_1$
- moments about RH end $\frac{mg}{2} = 3kx_3 + kx_2$

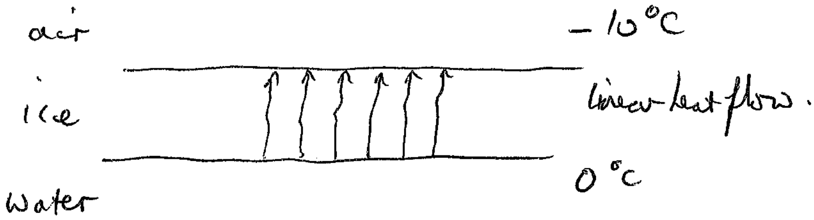
which will give $x_1 = 3x_3$

- and the Beam is linear (2).

or
- Moments about one end
- moments about centre
- Beam is linear

or
- total load
- moments about one end
- Beam is linear

The fact that the beam is linear (2) is required in all derivations.

(r) 

(15)

$$P = \frac{\Delta Q}{\Delta t} = +\lambda A\frac{(T_H - T_c)}{x} \quad (1)$$

Now, $P = \frac{mL}{t}$

$$= \rho A \frac{\Delta x}{\Delta t} . L \quad (2)$$

Equating power flow given by (1) and (2)

$$\rho \cancel{A} L \frac{\Delta x}{\Delta t} = \lambda \cancel{A} \frac{(T_H - T_c)}{x}$$

x is a finite thickness.

$$\rho L x \,\Delta x = \lambda (T_H - T_c)\Delta t .$$

lim of $\Delta x \to 0$, $\Delta t \to 0$.

$$L\rho\int_{x_1}^{x_2} x\,dx = \lambda(T_H - T_c)\int_{t_1}^{t_2} dt$$

$$L\rho\frac{1}{2}(x_2^2 - x_1^2) = \lambda(T_H - T_c)(t_2 - t_1)$$

$$t_2 - t_1 = \frac{L\rho}{2\lambda}\cdot\frac{(x_2^2 - x_1^2)}{(T_H - T_c)}$$

$$= \frac{330\times10^3\times900}{2\times2.1}\frac{((10\times10^{-2})^2 - (5\times10^{-2})^2)}{(0 - -10)}$$

$$= \frac{330\times10^3\times900}{4.2\times10}(75\times10^{-4})$$

$$= 53.0\times10^3\ s$$

= 884 minutes

= 14.7 hours.

(7)

Qu 2 (a) $f = 50H$, $I_{rms} = 10 \cdot 0A$, diameter $= 1mm$
free electrons cm^{-3} is $9 \cdot 0 \times 10^{22}$

$I_{max} = nAve$ $\qquad I_{max} = \sqrt{2} \cdot 10$

$10\sqrt{2} = 9 \times 10^{22} \times 10^{+6} \times \pi \left(\frac{10^{-3}}{2}\right)^2 \times v \times 1.6 \times 10^{-19}$

$$v_{max} = \frac{10\sqrt{2}}{9 \times 10^{28}\ \pi\ 25 \times 10^{-8} \times 1.6 \times 10^{-19}}$$

$\underline{v_{max} = 1.25 \times 10^{-3}\ ms^{-1}}$

AC $\quad v = v_0 \cdot \sin(\omega t)$ or $\cos(\omega t + \phi)$, etc.

$= A\omega \sin(\omega t)$

$v_{max} = A\omega$

$A = \frac{v_{max}}{2\pi f} \qquad \omega = 2\pi f.$

$= \frac{1.25 \times 10^{-3}}{2\pi 50}$

$\underline{A = 3.98 \times 10^{-6}\ m}$

(6)

(b) $R = \rho \frac{l}{A}$, $R' = \rho \frac{l'}{A'}$

With V constant $\Rightarrow A'l' = Al.$

Require $\frac{\delta R}{R} = \frac{R' - R}{R} = \left(\rho \frac{l'}{A'} - \rho \frac{l}{A}\right) / \rho \frac{l}{A}$

$= \frac{l'}{l} \frac{A}{A'} - 1$

$= \frac{l'}{l} \cdot \frac{A}{(Al/l')} - 1 = \frac{l'^2}{l^2} - 1$

But $l' = l + \epsilon$, so $\frac{\delta R}{R} = \frac{(l^2 + 2\epsilon l + \epsilon^2)}{l^2} - 1$

$$\frac{\delta R}{R} = \frac{2\varepsilon}{l} + \frac{\varepsilon^2}{l^2} \qquad \text{neglect 2nd term. in } \frac{\varepsilon^2}{l^2}$$

$$\text{So } \frac{\delta R}{R} \simeq \frac{2\varepsilon}{l}$$

$$\text{Hence } \delta R = 2 \times 0.001 \times 100.0$$
$$= 0.2\,\Omega$$
$$\text{and } \underline{\underline{R' = 100.2\,\Omega}}$$

(6)

Or for example,

$$R = \frac{\rho l}{A}$$

$$\delta R = \rho \frac{\delta l}{A} - \rho\, l \frac{\delta A}{A^2}$$

$$V = Al$$
$$\delta V = 0 = A\delta l + \delta A l$$
$$\text{so that } \delta A = -A\frac{\delta l}{l}$$

$$\text{Hence } \delta R = \rho\frac{\delta l}{A} + \frac{\rho l}{A^2}\cdot\left(A\frac{\delta l}{l}\right)$$
$$= \rho\frac{\delta l}{A} + \rho\frac{\delta l}{A}$$
$$= 2\rho\frac{\delta l}{A}$$
$$= 2 . R . \frac{\delta l}{l}$$
$$= 2R . \varepsilon/l.$$
$$= \underline{\underline{0.2\,\Omega}}$$

$$\underline{R' = 100.2\,\Omega}$$

18

(c)

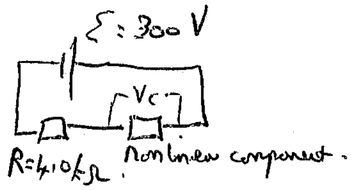


Diagram or indication of the correct circuit used ✓

$$\mathcal{E} = IR + V_c$$ ✓

$$\text{and } I = AV_c + BV_c^2$$

$$\mathcal{E} = RAV_c + RBV_c^2 + V_c$$ ✓

$$\mathcal{E} = (RA+1)V_c + RBV_c^2$$

quadratic in V_c with other terms given

$$RBV_c^2 + (RA+1)V_c - \mathcal{E} = 0$$ ✓

$$V_c = \frac{-(RA+1) \pm \sqrt{(RA+1)^2 + 4R.B.\mathcal{E}}}{2RB}$$

$$= \frac{-(0.28+1) \pm \sqrt{[1.28^2 + 4.4.0.005 \times 300]}}{2 \times 4 \times 0.005}$$

(A, B given in mA).

$$= \frac{-1.28 \pm \sqrt{1.28^2 + 24}}{0.04}$$

$$= \underline{\underline{94.6 \text{ V}}}$$ (take + sign). ✓

$$I = AV_c + BV_c^2$$

$$= 0.070 \times 94.6 + 0.0050 \times 94.6^2$$

$$= \underline{51.4 \text{ mA}}$$ ✓

[a check: $300\text{V} - 4000 \times 51.35 \times 10^{-3}\text{V} = 94.586\text{V}$].

$$\mathcal{E} - R \quad I = V_c$$

(6)

part (c) can also be done directly in terms of I, although this then requires a calculation of V_c rather than obtaining an expression.

From Kirchoff II, $V_c = 300 - 4.0 \times 10^3 I$

and with $I = AV_c + BV_c^2$

We have,

$$I = 0.07\times10^{-3}(300 - 4\times10^3 I) + 0.005\times10^{-3}(300 - 4\times10^3 I)^2$$

$$I = 0.021 - 0.28I + 0.005\times10^{-3}(300^2 + 16\times10^6 I^2 - 2.4\times10^6 I)$$

$$I = 0.021 - 0.28I + 0.45 + 80I^2 - 12I$$

$$I = 0.\cancel{47} - 12.28I + 80I^2$$

so that $80I^2 - 13.28I + 0.\cancel{47} = 0$

$$I = \frac{13.28 \pm 5.09}{160} \times 1000 \text{ mA}.$$

$$= 114.8 \text{ mA or } 51.2 \text{ mA}.$$

Max current, $I_{max} = \dfrac{300 \text{ V}}{4000\,\Omega} \times 1000 \text{ mA}$

$= 75 \text{ mA}.$

$\therefore$ $I = 51.2$ mA
and $V_c = 95.3$ V. — some slight rounding errors evident.

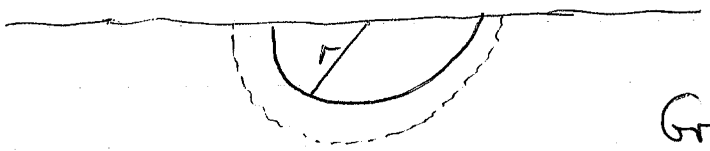
$I = 51$ mA

$V_c = 95$ V.

✓

20

(d)



$r = 0{\cdot}10\ \text{m}$

Grasp of the idea by words or diagram.

(i) $R = \dfrac{\rho l}{A}$ $\quad A = 2\pi r^2$

$= \dfrac{\rho \cdot \delta r}{2\pi r^2}$

[For $\delta r << r$, $R = \dfrac{60 \times \delta R}{2\pi (0{\cdot}10)^2}$

$R = 95 \times \delta R \ (\Omega)$ } One mark if this approach taken.

(ii) In general $R = \dfrac{\rho}{2\pi}\displaystyle\int_r^\infty \dfrac{dr}{r^2}$

$= -\dfrac{\rho}{2\pi}\left[\dfrac{1}{r}\right]_r^\infty$

$= \dfrac{\rho}{2\pi}\left[\dfrac{1}{r}\right]_\infty^r$

$= \dfrac{\rho}{2\pi}\,\dfrac{1}{r}$

$\therefore \ R = \dfrac{60}{2\pi \cdot 0{\cdot}1}$

$= \dfrac{300}{\pi} = 95{\cdot}5$

$= \underline{\underline{95\ \Omega}}$

(7)

Qu: 3

A mark for each step, up to 9 max. Candidates may calculate in a different order to this.

Once the tug has slowed sufficiently, its thrust of 35 kN will not allow the 50 kN brake to release any more of the cable.

Neglecting any forces due to the water flowing around the tug and barge,

At moment of impact, and after, force to accelerate the barge is 50 kN

Acceleration of barge is $\frac{50 \times 10^3}{6300 \times 10^3}$

$$a_b = \frac{1}{126} \ ms^{-2}$$

Force on the tug forwards is $(35 - 50) \times 10^3 N$

$$= -15 \times 10^3 N$$

acceleration of tug, $a_t = \frac{-15 \times 10^3}{450 \times 10^3}$

$$= \frac{1}{30} \ ms^{-2}$$

These accelerations are constant for a time t.

$\therefore$ using "$V = u + at$"

$$V_{barge} = 0 + \frac{1}{126} t$$

and

$$V_{tug} = 2.5 - \frac{1}{30} \cdot t$$

When $V_{tug} = V_{barge}$ then the cable is no longer slipping.

$$\therefore \frac{1}{126} t = 2.5 - \frac{1}{30} \cdot t$$

$$\underline{t = 60.6 \ s}$$

Then $V_{constant} = \frac{60.6}{126} = \underline{\underline{0.48 \ ms^{-1}}}$

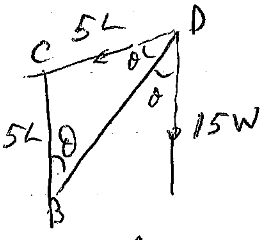
Consequently, $s = ut + \frac{1}{2}at^2 \Rightarrow S = S_{tug} - S_{barge}$

$$S = S_{tug} - S_{barge}$$
$$= 2\cdot5 \times 60\cdot6 - \frac{1}{2}\cdot\frac{1}{30}(60\cdot6)^2 - 0 - \frac{1}{2}\frac{1}{126}(60\cdot6)^2$$
$$= 75\cdot7 \text{ m}$$
$$= \underline{\underline{76 \text{ m}}}$$

(9)

$\boxed{\text{Various Methods} - \text{2 marks for method, 1 mark for result}}$

(b)



Since CB is vertical, and the cable is vertical, the angle CBD is equal to the angle between the cable and BD.

[Let $\angle CDB = \theta$]. Also, CB = CD, so $\angle CDB = \angle CBD$

so BD bisects the angle at the top.

$\boxed{\checkmark\checkmark \text{ Working}}$

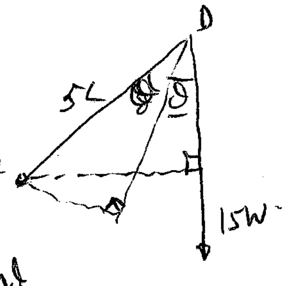
∴ Since BD can provide no torque,

$$15W\cos\theta = T_{CD}\cdot\cos\theta$$
$$\underline{T_{CD} = 15\,W}$$ ✓ result.

Or using cosine rule, $(5L)^2 = (8L)^2 + (5L)^2 - 2.5L.8L.\cos\theta$

$$64 = 80\cos\theta$$
$$\cos\theta = 0\cdot8$$

Take moments about C



$$0 = T_{BD}\cdot 5L.\sin\theta - 15W.5L\sin 2\theta$$

i.e. $T_{BD}.\cancel{5L}.\cancel{\sin\theta} = 15W.\cancel{5L}.2\cancel{\sin\theta}.\cos\theta$

$\boxed{\checkmark\checkmark \text{ Working}}$

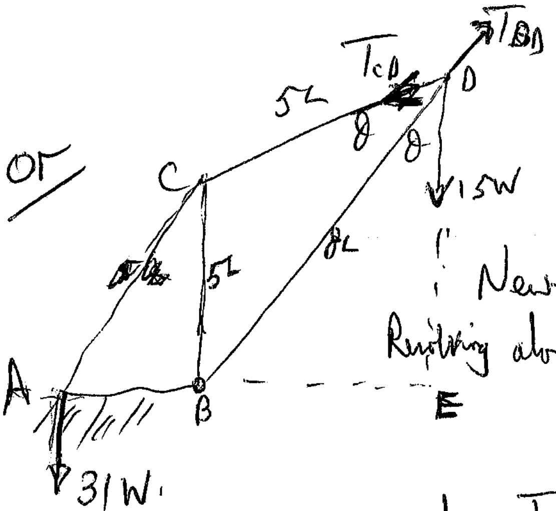
$$T_{BD} = 15W \times 2 \times 0\cdot8$$
$$= \underline{\underline{24W}}$$

✓ result.

Resolve forces on D vertically: $15W = 24W.\cos\theta - T_{CD} \times \cos 2\theta$

$$15W = 24W \times 0\cdot8 - T_{CD} \times 0\cdot28$$
$$\underline{\underline{T_{CD} = 15W}}$$

Or



Newton I. and if it is shown that BD bisects the angle,
Resolving along DB. $T_{CD} . \sin\theta = 15W . \sin\theta$

$\underline{T_{CD} = 15W}$.

and $T_{BD} = T_{CD} \cos\theta + 15W \cos\theta$
$= 2 \times 15W \times 0.8$
$= \underline{\underline{24W}}$.

(iii) <u>Distance AB</u>.

Take moments about B.

$\cos\theta = 0.8$
$\sin\theta = 0.6$.

$31W \times AB = 15W \times BE$

$31W \times AB = 15W \times 8L \sin\theta$

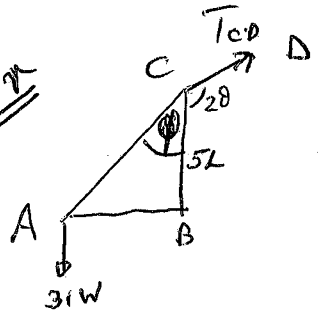
$AB = \frac{15}{31} \times 8L \times 0.6 \quad = 2.32L$.

working + result.
✓ ✓

$\underline{\underline{AB = 2.3L}}$

(8)

Or



Resolving horizontal forces on C.
$T_{CD} . \cos(90 - 2\theta) = T_{CA} \sin\phi$

Resolving vertically at A.
$31W = T_{CA} \cos\phi$

Eliminating T_{CA}:

$$\frac{31W}{\cos\phi} . \sin\phi = 15W \cos(90 - 2\theta)$$

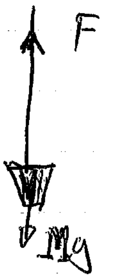
i.e. $\tan\phi = \frac{15}{31} . \sin 2\theta \qquad \cos(90 - 2\theta) = \sin 2\theta$

So, $\tan\phi = \frac{15}{31} . 2\sin\theta . \cos\theta = \frac{15}{31} . 2 \times 0.6 \times 0.8 = 0.4645$

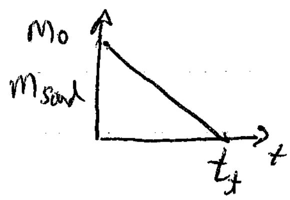
and $\tan\phi = \frac{AB}{5L} \Rightarrow AB = 5L \tan\phi = \underline{\underline{2.32L}}$

(24)

(c)



[M = Mass of bucket
m_0 = initial mass of sand]



(t_f = time for sand to leak out.)

$$m_s = m_0 - kt$$
$$= m_0 - \frac{m_0}{t_s} \cdot t$$
$$= m_0\left(1 - \frac{t}{t_f}\right)$$

total mass lifted is, $\mathcal{M} = M + m_0\left(1 - \frac{t}{t_f}\right)$

Newton II (upwards) $\mathcal{M} a = F - \mathcal{M} g$

$$a = \frac{F}{\mathcal{M}} - g$$

$$\int_0^t a \, dt = \int_0^V dV = V$$

$$V = \int_0^{t_f} \left(\frac{F}{M + m_0\left(1 - \frac{t}{t_f}\right)} - g\right) dt$$

$$V = \int_0^{t_f} \frac{F}{\left(M + m_0 - \frac{m_0}{t_f} \cdot t\right)} - \int_0^{t_f} g \, dt$$

$$V = -\frac{t_f}{m_0} F \left[\ln\left(M + m_0 - \frac{m_0}{t_f} \cdot t_f\right) - \ln(M + m_0)\right] - g t_f$$

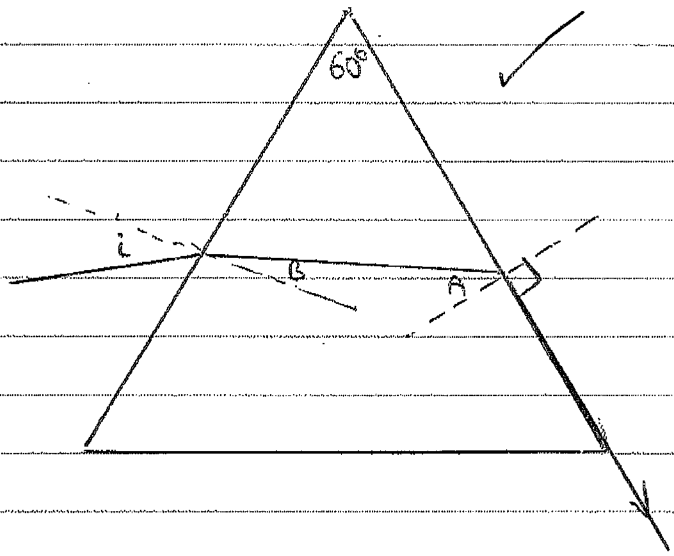
$$V = -\frac{t_f}{m_0} \cdot F \ln\left(\frac{M}{M + m_0}\right) - g t_f$$

$$V = \frac{t_f}{m_0} \cdot F \cdot \ln\left(1 + \frac{m_0}{M}\right) - g \cdot t_f.$$

(8)

25

Q4. a)



$\sin A = \frac{1}{1.5}$

$A = 41.81°$

$A + B = 60° \rightarrow B = 18.19°$

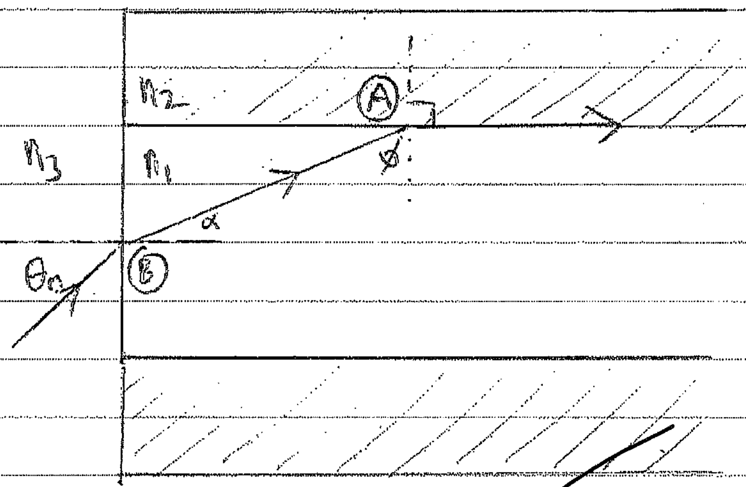
$\frac{\sin i}{\sin 18.19°} = 1.5$

$i = 27.92°$

(Greatest) angle, $i = 28°$

(5)

b)



Suitable angles marked on a diagram

At (A) $n_1 \sin\phi = n_2 \sin 90° \rightarrow \sin\phi = \frac{n_2}{n_1} \rightarrow \phi = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

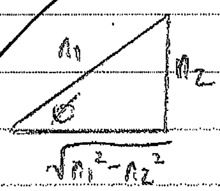
$\alpha = 90 - \phi = 90 - \sin^{-1}\left(\frac{n_2}{n_1}\right)$

$n_3 \sin\theta_M = n_1 \sin\alpha = n_1 \sin\left(90 - \sin^{-1}\left(\frac{n_2}{n_1}\right)\right)$

$= n_1 \cos\left(\sin^{-1}\left(\frac{n_2}{n_1}\right)\right)$

$= n_1 \cdot \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$

$= \sqrt{n_1^2 - n_2^2}$

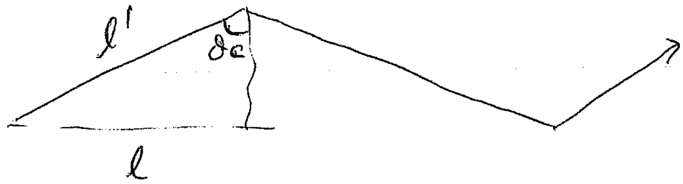


$\sin\theta_M = \frac{\sqrt{n_1^2 - n_2^2}}{n_3}$

$\left[\theta_M = \sin^{-1}\left(\frac{\sqrt{n_1^2 - n_2^2}}{n_3}\right)\right]$ or $\sin\theta_M = \frac{n_1}{n_3}\sqrt{1 - \frac{n_2^2}{n_1^2}}$

(4)

(ii)



$$t_l = \frac{n_1 l}{c} = 4.953 \times 10^{-4}\,s$$

$$t_l' = \frac{n_1 l}{c \sin\theta_c} = 5.053 \times 10^{-4}\,s.$$

$$\sin\theta_c = 0.9802$$

$$\Delta t = 0.100 \times 10^{-4}\,s$$
$$= 1.00 \times 10^{-5}\,s$$
$$= \underline{10\,\mu s}$$

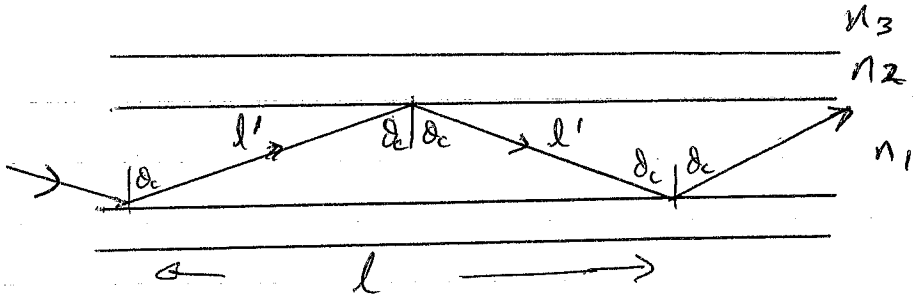
(iii) $$f_{max} = \frac{1}{\Delta t} = \underline{99.9\ kHz}.$$

So that pulses are not overlapping.

(4)

parts (ii) (iii) and (iv) can be done in any order. Some will find that doing the calculation in (ii) first helps them with the algebra required in part (iv).

(iv)



In the limit of $\theta \to \theta_m$, the angle of incidence in the fibre $\to \theta_c$

2 marks for the idea of the two paths

Light travelling along the core travels a distance l in time $t_{min} = \frac{n_1 l}{c}$ ✓

" " " " l' travels distance l' in $t_{max} = \frac{n_1 l}{c \sin\theta_c}$ ✓

since $l' = \frac{l}{\sin\theta_c}$

and $n_1 \sin\theta_c = n_2$

So $l' = l\frac{n_1}{n_2}$ ✓

and $t_{max} = \frac{n_1^2}{n_2}\frac{l}{c}$

So $\Delta t = \frac{n_1 l}{c}\left(\frac{n_1}{n_2} - 1\right)$

$= \frac{l}{c'}\frac{\Delta n}{n_2}$ ✓

4

Total. (12) i.e. [4+4+4]

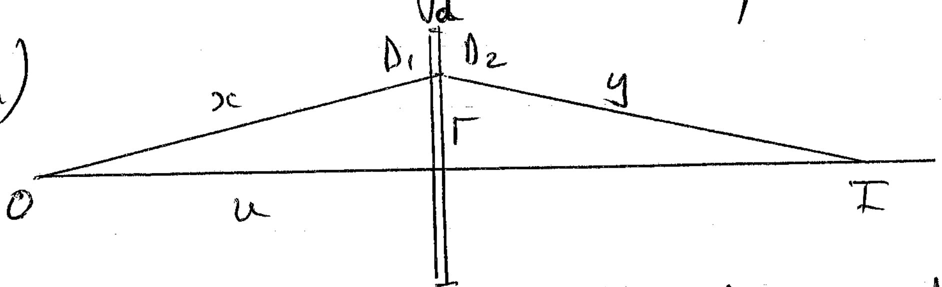
(c) for $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$\lim_{u\to\infty}$ is $v=f$

So as the object distance $\to\infty$, rays of light travelling ... lens "become parallel" to the axis (2)

and so f is the distance/point from the lens at which rays from a distant object will cross/focus (on the principal axis).

(d)



We need to compare the time along the axial path OI and the path OD_1, D_1D_2, D_2I.

Time from O to I along the axis, $t_1 = \frac{u}{c}+\frac{dn_0}{c}+\frac{v}{c}$

Time along OD_1, D_1D_2, D_2I is $t_2 = \frac{x}{c}+\frac{dn_0(1-kr^2)}{c}+\frac{y}{c}$

Now $x^2 = r^2+u^2$
$= u^2\left(1+\frac{r^2}{u^2}\right)$

$x \approx u\left(1+\frac{1}{2}\frac{r^2}{u^2}\right)$

Similarly $y \approx v\left(1+\frac{1}{2}\frac{r^2}{v^2}\right)$

Hence $t_2 = \frac{u}{c}\left(1+\frac{1}{2}\frac{r^2}{u^2}\right)+\frac{dn_0}{c}(1-kr^2)+\frac{v}{c}\left(1+\frac{1}{2}\frac{r^2}{v^2}\right)$

For $t_1 = t_2$

$\cancel{\frac{u}{c}}+\cancel{\frac{dn_0}{c}}+\cancel{\frac{v}{c}} = \cancel{\frac{u}{c}}+\frac{1}{2c}\frac{r^2}{u}+\cancel{\frac{dn_0}{c}}-\frac{dn_0kr^2}{c}+\cancel{\frac{v}{c}}+\frac{1}{2c}\frac{r^2}{v}$

which gives $$0 = \frac{1}{2c}\frac{r^2}{u} - \frac{d n_0 k r^2}{c} + \frac{1}{2c}\frac{r^2}{v}$$

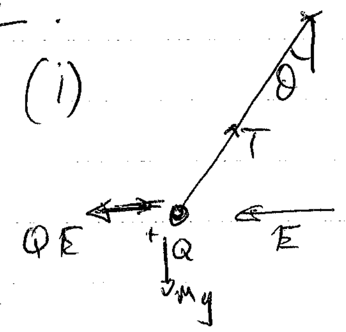
$$2 d n_0 k = \frac{1}{u} + \frac{1}{v}$$

with $2 d n_0 k$ constant
and $f = \dfrac{1}{2 d n_0 k}$.

(6)

Qu 5

(a) (i)

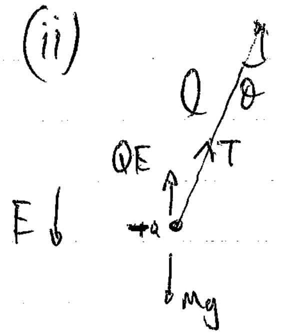


Resolve forces ↓ $Mg - T\cos\theta = 0$ ✓

← $QE - T\sin\theta = 0$ ✓

$$\frac{QE}{Mg} = \tan\theta$$ ✓

(ii)



Not in equilibrium.

Resolve ⊥ to string:

→ $-QE\sin(-\theta) + Mg\sin(-\theta) = Ma$

and $a = l\ddot{\theta}$

∴ $QE\sin\theta - Mg\sin\theta = Ml\ddot{\theta}$ ✓ or (equivalent)

for small θ, $\sin\theta \approx \theta$

Hence $(QE - Mg)\theta = Ml\ddot{\theta}$

or $-(Mg - QE)\theta = Ml\ddot{\theta}$

SHM.

$$\omega^2 = \frac{(Mg - QE)}{Ml}$$

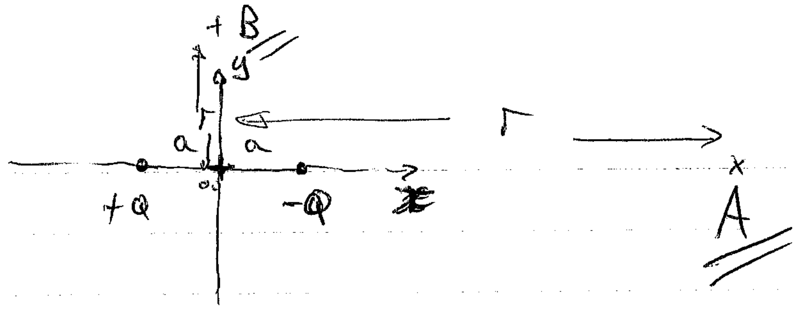
and $$T = 2\pi\sqrt{\frac{Ml}{(Mg - QE)}}$$ ✓

(iii) Oscillations will occur when $QE < Mg$ ✓

(field is downwards, charge is negative so electric force is upwards).

(6)

(b)



(i) for point A:

$$V_A = \frac{1}{4\pi\epsilon_0} \cdot Q \left[\frac{1}{r+a} - \frac{1}{r-a} \right]$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r} \left[\frac{1}{1+a/r} - \frac{1}{1-a/r} \right]$$

$$\approx \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r} \left[1 - \frac{a}{r} - \left(1 + \frac{a}{r}\right) \right]$$

$$= -\frac{1}{4\pi\epsilon_0} \frac{2Qa}{r^2}$$

If the charges are $+Q \leftrightarrow -Q$, then

$$V \simeq + \frac{1}{4\pi\epsilon_0} \cdot \frac{2Qa}{r^2}$$

(Either way is O.K.)

or The binomial approximation may not be used;

then $$V_A = \frac{1}{4\pi\epsilon_0} Q \left[\frac{(r-a) - (r+a)}{(r+a)(r-a)} \right]$$

$$= \frac{1}{4\pi\epsilon_0} Q \frac{2a}{(r^2 - a^2)}$$

$$= \frac{1}{4\pi\epsilon_0} Q \frac{2a}{r^2} \quad \text{to first order in } \frac{a}{r}$$

NB. Solution may have $k = \frac{1}{4\pi\epsilon_0}$. This is O.K.

For $\underline{A}$,

$$E = \frac{1}{4\pi\varepsilon_0} Q \left\{ \frac{1}{(r+a)^2} - \frac{1}{(r-a)^2} \right\}.$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \left\{ \frac{1}{(1+\frac{a}{r})^2} - \frac{1}{(1-\frac{a}{r})^2} \right\}.$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \left\{ 1 - \frac{2a}{r} - \left(1 + \frac{2a}{r}\right) \right\}.$$

$$= (-) \frac{1}{4\pi\varepsilon_0} Q \cdot \frac{4a}{r^3}$$

only magnitude required, so ignore any sign.
(It does not matter if it is included)

~~OR~~

$$E = \frac{1}{4\pi\varepsilon_0} Q \left\{ \frac{(r-a)^2 - (r+a)^2}{(r+a)^2(r-a)^2} \right\}$$

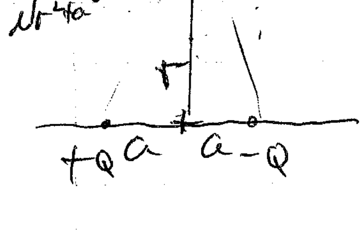
$$= \frac{1}{4\pi\varepsilon_0} Q \left\{ \frac{\cancel{r^2} + \cancel{a^2} - 2ra - (\cancel{r^2} + \cancel{a^2} + 2ra)}{(r^2-a^2)^2} \right\}$$

$$= \frac{1}{4\pi\varepsilon_0} Q (-) \frac{4ra}{r^4 (1 - \frac{a^2}{r^2})^2}$$

$$\simeq \cancel{(-)} \frac{1}{4\pi\varepsilon_0} Q \frac{4a}{r^3}$$ to first order in $\frac{r}{a}$.

For B

$$V = \frac{1}{4\pi\varepsilon_0} \cdot Q \left\{ \frac{1}{\sqrt{r^2+a^2}} - \frac{1}{\sqrt{r^2+a^2}} \right\} = 0$$



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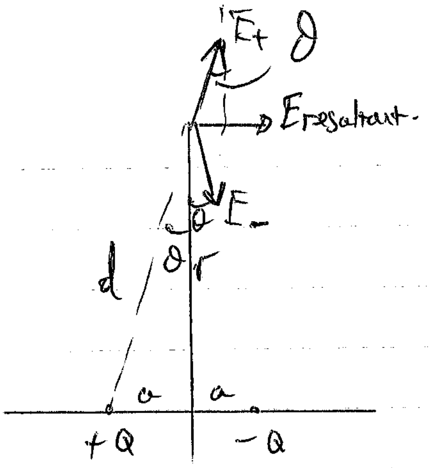


Components of $\vec{E}$ perpendicular to the line joining the charges cancel.

Eresultant is parallel to the line joining the charges from + to − ✓

$$E_{res} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{d^2}\sin\theta + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{d^2}\sin\theta \quad ✓$$

$$= \frac{2Q}{4\pi\varepsilon_0}\cdot\frac{1}{(a^2+r^2)}\cdot\frac{a}{\sqrt{a^2+r^2}}$$

$$= \frac{2Qa}{4\pi\varepsilon_0}\cdot\frac{1}{(a^2+r^2)^{\frac{3}{2}}}$$

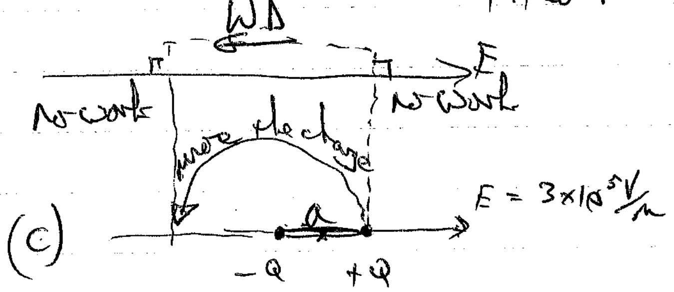
$$= \frac{2Qa}{4\pi\varepsilon_0 r^3\left(1+\frac{a^2}{r^2}\right)^{\frac{3}{2}}}$$

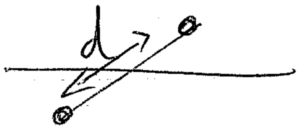
$$\approx \frac{2Qa}{4\pi\varepsilon_0 r^3}\left(1-\frac{3}{2}\frac{a^2}{r^2}\right).$$

$$= \frac{2Qa}{4\pi\varepsilon_0 r^3} \quad \text{to lowest order} \quad ✓$$

(10)

(c)





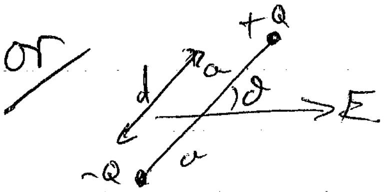
Keep one charge fixed and move the other charge (+) against the field. Can move on a virtual path ⊥ to E, antiparallel to E by 2d, and then ⊥ to E.

WD = force × displacement along the field
$= QE \times 2d$
$= 2QdE$ (if work done by the field a (−) sign appears. — ignore any sign)

Method ✓

Answer ✓ $W = 1{\cdot}9(2)\times10^{-24}\,\mathrm{J}$

(2)

or 

$$\text{torque on dipole} = -QE\frac{d}{2}.\sin\theta - QE\frac{d}{2}\sin\theta$$

$$= -QEd\sin\theta$$

Work done on the dipole is $QEd\sin\theta.d\theta$ to rotate it through $d\theta$

$$\therefore WD = \int_0^{\pi} QEd\sin\theta\, d\theta$$

$$= QEd\left[-\cos\theta\right]_0^{\pi}$$

$$= -QEd(-1-1)$$

$$= \underline{2QEd}$$

So, $W = 1.92 \times 10^{-24}\,J$

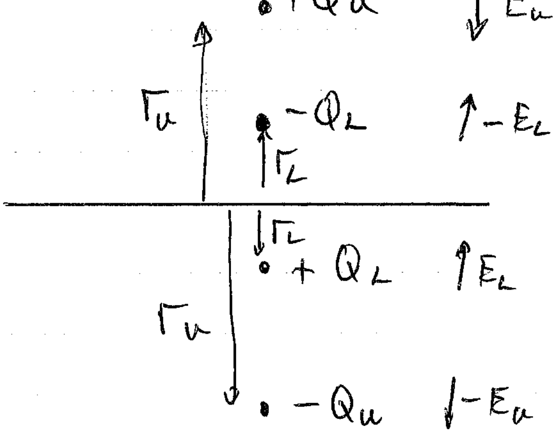
(d) upper cylinder. $Q_u = n.\pi r^2 h.$

$$= 0.21\times10^{-9}\times\pi\times5000^2\times3000$$

$$= \underline{49.5\,C.}$$ ✓

Lower cylinder $Q_L = (-)0.18\times10^{-9}\times\pi\times5000^2\times7000$

$$= \underline{(-)\ 99.0\,C}$$ ✓



$r_u = 10.5\,km$

$r_L = 5.5\,km$

35

Field at the surface, $E_s = \frac{1}{4\pi\varepsilon_0}\left\{ +\frac{Q_u}{r_u^2} - \frac{Q_L}{r_L^2} - \frac{Q_L}{r_L^2} + \frac{Q_u}{r_u^2} \right\}$.

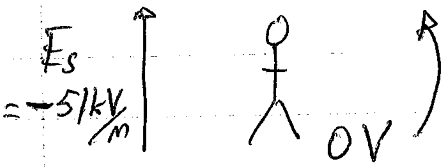
Sign: ↓ down is positive field.

$$E_s = \frac{1}{4\pi\varepsilon_0}\left\{ 2\frac{Q_u}{r_u^2} - 2\frac{Q_L}{r_L^2} \right\}$$

$$E_s = 9\times10^9 \times 2 \left\{ \frac{49.5}{10500^2} - \frac{99.0}{5500^2} \right\}$$

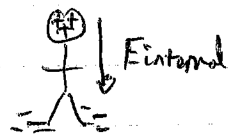
$$\underline{E_s = (-)50.8 \text{ kV m}^{-1}}$$

-102 kV.



The potential at the head is $\underline{\underline{-102\text{ kV}}}$.

As the person is a conductor, it may be illustrated as



since there must be an opposing field inside to prevent a flow of charge

So answer may be given as head is more positive than the feet.

(7)

Question 6

(a) If source stationary, $f\Delta t$ waves emitted in Δt.

Boat receives $f\Delta t$ waves + $\frac{u\Delta t}{\lambda_s}$ waves due to its movement.

$$\therefore \quad f'\Delta t = f\Delta t + \frac{u\Delta t}{\lambda_s}$$

with $f \cdot \lambda_s = c$

$$f' = f + \frac{u}{c/f}$$

$$f' = f\left(1 + \frac{u}{c}\right) = f\left(\frac{u+c}{c}\right)$$. (No mark for the answer).

or relative velocity between the waves and the boat moving towards them is $u + c$

so $f' = \frac{(u+c)}{\lambda}$ $\quad f \cdot \lambda = c$

$$= \left(\frac{u+c}{c}\right)f$$

or time taken for boat to travel from crest to crest,

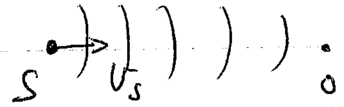
$$\Delta t = \frac{\text{distance}}{\text{relative speed}} = \frac{\lambda}{u+c} = \frac{c}{f(u+c)}$$

$$\therefore \quad f = f\frac{(u+c)}{c}.$$

Any method will do, but it must be clear to the marker. A set of mixed up symbols and the given answer will NOT do!

(4)

(b) (i)



In time Δt, $f \cdot \Delta t$ waves emitted by source.
These fit in a length $(c-u) \cdot \Delta t$

$$\therefore \quad \lambda = \frac{(c-u)\Delta t}{f \;\; \Delta t} = \frac{c-u}{f}$$

But this shorter λ is detected by the observer as a high frequency

$$\frac{c}{f'} = \frac{(c-v_s)}{f}$$

so

$$f' = \frac{f}{\left(1 - \frac{v_s}{c}\right)}$$

Show that must be a derivation. No marks for the answer itself, as it is given in the question.

(ii) Source moves away $v_s \longrightarrow -v_s$.

$$v_s = c/2$$

$$f' = \frac{f}{1+\frac{1}{2}}$$

$$f' = \frac{2}{3}f$$

$$\therefore \quad \Delta f = \frac{2}{3}f - f = -\frac{1}{3}f$$

Value and lower frequency or the sign.

(iii) approx: $f' \approx f\left(1 + \frac{V_s}{c}\right)$

so $f' - f = f\frac{V_s}{c}$.

$$\frac{\Delta f}{f} \approx \frac{V_s}{c}$$

(6)

or $f'\left(1-\frac{V_s}{c}\right) = f$

$$f' - f'\frac{V_s}{c} = f$$

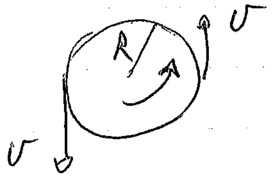
$$f' - f = f'\frac{V_s}{c}$$

$$f' - f = f\left(\frac{V_s}{c}\right)\frac{1}{1-\frac{V_s}{c}}$$

$$\simeq f\frac{V_s}{c}$$

$$\frac{\Delta f}{f} \simeq \frac{V_s}{c}.$$

(c) (i)



$v = \frac{2\pi R}{T}$ ✓

for $v \ll c$ speed of light. stated approx ✓

$$\frac{\delta f}{f} \simeq \frac{v}{c}$$

$$\underline{\delta f = \frac{2\pi R}{T}\cdot\frac{f}{c}}$$

(ii)

$$\frac{\delta f}{f} = \frac{2\pi}{T}\cdot\frac{R}{c}$$

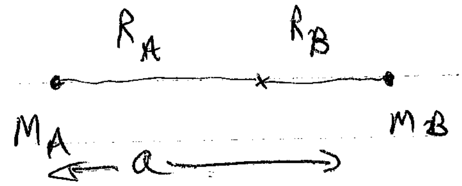
$\frac{M}{\rho} = \frac{4}{3}\pi R^3$ ✓

$R = 699\times10^6$ m. ✓

$$\underline{\frac{\delta f}{f} = 6.9\times10^{-6}}$$ ✓

(5)

(d) (i)



The two stars have the same angular velocity, ω.

Centripetal force about C of M is provided by gravity.

$$\therefore \quad M_A R_A \omega^2 = \frac{G M_A M_B}{a^2}$$ ✓

$$\text{and} \quad M_B R_B \omega^2 = \frac{G M_A M_B}{a^2}$$

$$\therefore \quad R_A \omega^2 = \frac{G M_B}{a^2}$$

$$\text{and} \quad R_B \omega^2 = \frac{G M_A}{a^2}$$ ✓

$$\text{Adding} \quad \omega^2 (R_A + R_B) = \frac{G}{a^2}(M_A + M_B)$$ ✓

$$\omega^2 a = \frac{G}{a^2} . M$$

$$\frac{4\pi^2}{T^2} = \frac{GM}{a^3}$$

$$\text{or} \quad T^2 = \frac{4\pi^2}{GM} . a^3$$

Do not allow a straight $\frac{mv^2}{R} = \frac{GmM}{R^2}$ here to get Kepler III for a central orbit

(3)

(ii)

$$M = \frac{4\pi^2 a^3}{GT^2}$$

$$= \frac{4\pi^2 . (19.8 \times 1.5 \times 10^{11})^3}{6.67 \times 10^{-11} \times (50.1 \times 365 \times 24 \times 3600)^2}$$

$$= 6.21 \times 10^{30} \text{ kg}$$ ✓

(1)

(40)

(iii) Centripetal force due to gravity

$$\frac{M_A V_A^2}{R_A} = \frac{G M_A . M_B}{a^2}$$

Since $V_A = \frac{2\pi R_A}{T}$

then $$\frac{V_A}{\cancel{R_A}} . \frac{2\pi \cancel{R_A}}{T} = \frac{G M_B}{a^2}$$

so $$V_A = \frac{G M_B . T}{2\pi a^2}$$

(2)

(iv) $$\frac{\Delta f}{f} = \frac{V_A}{c} = \frac{G M_B . T}{2\pi a^2 c}$$

so $$M_B = \frac{2\pi a^2 c . \Delta f}{f G T}$$

$$M_B = 5.65 \times 10^{30} \text{ kg}$$

Total mass found from $T^2 = \frac{4\pi^2}{GM} . a^3$ in part (ii)

to give $M_A = 6.2 \times 10^{30} - 5.65 \times 10^{30}$

$$M_A = 5.6 \times 10^{29} \text{ kg}$$

(3)

(v) No external torque, or in the zero momentum frame, etc.

$$M_A R_A = M_B R_B$$

$$\therefore \frac{R_A}{R_B} = \frac{M_B}{M_A}$$

$$= 10.(1)$$

(1)

/10